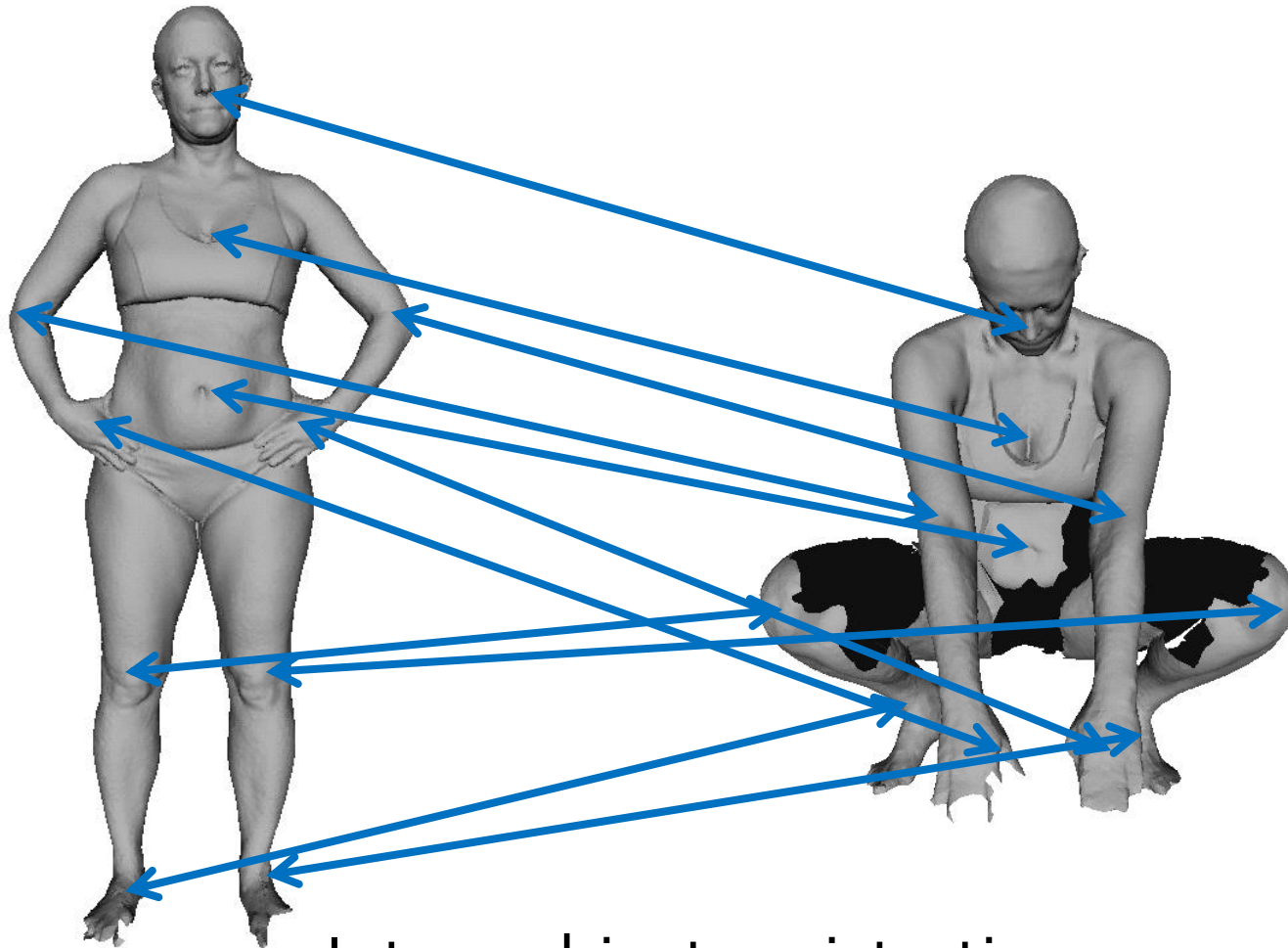


Robust Nonrigid Registration by Convex Optimization

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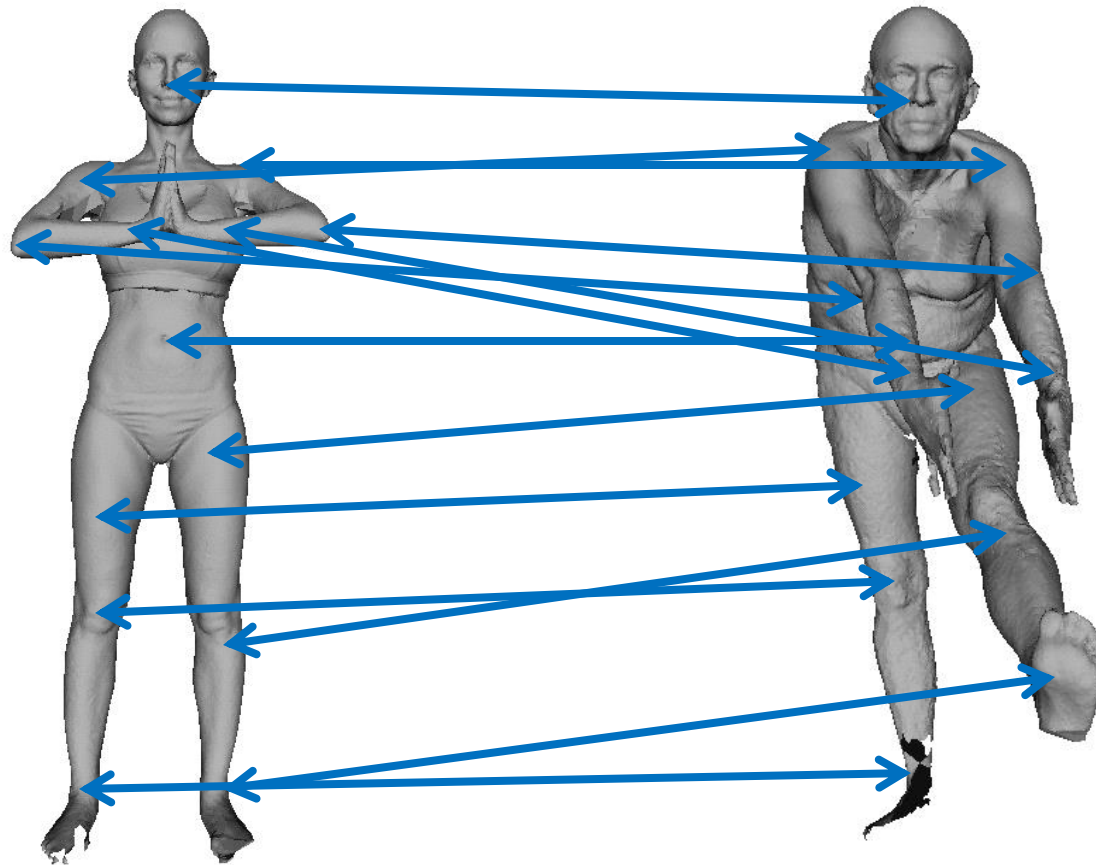
Vladlen Koltun
Intel Labs

Nonrigid Registration



Intra-subject registration

Nonrigid Registration



Inter-subject registration

Uses of Nonrigid Registration

- Loop closure in dynamic reconstruction
- Shape analysis
- Propagation of material properties across 3D models
- Surface completion

Prior work

- Intrinsic descriptors
 - Heat kernel signature [Sun et al. 2009]
 - Wave kernel signature [Aubry et al. 2011]
 - Global point signature [Rustamov 2007]
 - Spectral descriptors [Litman et al. 2014]
 - Optimal descriptors [Windheuser et al. 2014]

Prior work

- Generalized multidimensional scaling (GMDS)
[Bronstein et al. 2006]

- Given two surfaces $\mathcal{S}, \mathcal{T} \subset \mathbb{R}^3$
- Compute mapping $f : \mathcal{S} \rightarrow \mathcal{T}$
- Minimize highly nonconvex objective

$$E(f) = \sum_{i,j} w_{i,j} |d_{\mathcal{S}}(s_i, s_j) - d_{\mathcal{T}}(f(s_i), f(s_j))|^p \quad (\text{GMDS})$$

- Optimize $E(f)$ by gradient descent
- Easily stuck at bad local minima

Optimization

- Let $S = \{s_i\}_{i=1}^n$ and $T = \{t_j\}_{j=1}^m$ be points densely sampled over \mathcal{S} and \mathcal{T}
- Optimize labeling $l : S \rightarrow L$ (L is a set of m labels)

$$E(f) = \sum_{i,j} w_{i,j} |d_{\mathcal{S}}(s_i, s_j) - d_{\mathcal{T}}(f(s_i), f(s_j))|^p \quad (\text{GMDS})$$

Continuous Markov random field (MRF)



$$\underset{l}{\text{minimize}} \sum_{i,j} w_{i,j} |d_{\mathcal{S}}(s_i, s_j) - d_{\mathcal{T}}(t_{l_i}, t_{l_j})|^p \quad (\text{Discrete MRF})$$

Optimization

$$\underset{\mathbf{1}}{\text{minimize}} \sum_{i,j} w_{i,j} |d_{\mathcal{S}}(s_i, s_j) - d_{\mathcal{T}}(t_{l_i}, t_{l_j})|^p \quad (\text{Discrete MRF})$$



$$\underset{\mathbf{0} \leq \mathbf{x} \leq \mathbf{1}}{\text{minimize}} \sum_{i,j} \sum_{a,b} \theta_{ij}(a,b) x_{ij}^{ab} \quad (\text{Linear program})$$

$$\text{s.t.} \quad \sum_b x_{ij}^{ab} = x_i^a, \quad \forall i, j \in S, \forall a \in L$$

$$\sum_a x_i^a = 1, \quad \forall i \in S$$

$$\text{where } \theta_{ij}(l_i, l_j) = w_{i,j} |d_{\mathcal{S}}(s_i, s_j) - d_{\mathcal{T}}(t_{l_i}, t_{l_j})|^p$$

Optimization

$$\begin{array}{ll} \text{minimize} & \sum_{i,j} \sum_{a,b} \theta_{ij}(a,b) x_{ij}^{ab} \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} & \end{array} \quad \text{(Linear program)}$$

$$\text{s.t.} \quad \sum_b x_{ij}^{ab} = x_i^a, \quad \forall i, j \in S, \forall a \in L$$

$$\sum_a x_i^a = 1, \quad \forall i \in S$$



$$\begin{array}{ll} \text{maximize} & \sum_i \min_a \tilde{\theta}_i^a + \sum_{i,j} \min_{a,b} \tilde{\theta}_{ij}^{ab}, \\ \delta & \end{array} \quad \text{(Dual LP)}$$

$$\text{where} \quad \tilde{\theta}_i^a = \sum_j \delta_{ji}^a$$

$$\tilde{\theta}_{ij}^{ab} = \theta_{ij}(a,b) - \delta_{ji}^a - \delta_{ij}^b$$

Optimization

$$\underset{\mathbf{0} \leq \mathbf{x} \leq \mathbf{1}}{\text{minimize}} \quad \sum_{i,j} \sum_{a,b} \theta_{ij}(a,b) x_{ij}^{ab} \quad (\text{Linear program})$$

$$\text{s.t.} \quad \sum_b x_{ij}^{ab} = x_i^a, \quad \forall i, j \in S, \forall a \in L$$

$$\sum_a x_i^a = 1, \quad \forall i \in S$$



$$\underset{\delta}{\text{maximize}} \quad \sum_i \min_a \tilde{\theta}_i^a + \sum_{i,j} \min_{a,b} \tilde{\theta}_{ij}^{ab}, \quad (\text{Dual LP})$$

$$\text{where} \quad \tilde{\theta}_i^a = \sum_j \delta_{ji}^a \quad \text{TRW-S [Kolmogorov 2006]}$$

$$\tilde{\theta}_{ij}^{ab} = \theta_{ij}(a,b) - \delta_{ji}^a - \delta_{ij}^b$$

Objective

- Penalty

$$\rho(x, y) = \exp\left(-\frac{\min(x, y)}{b}\right) \min(|x - y|, \tau)$$

- Objective

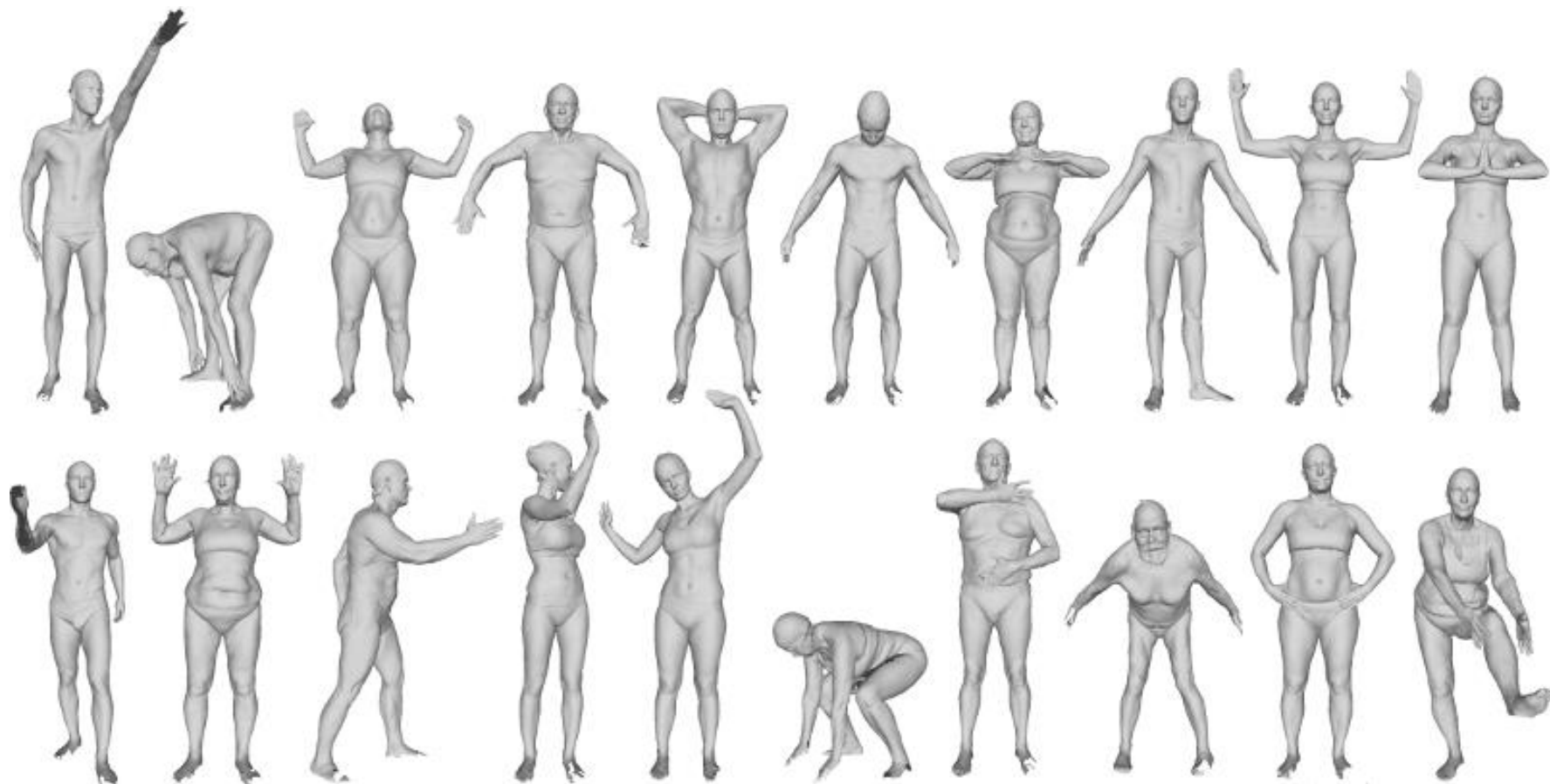
$$\begin{aligned} E(\mathbf{l}) &= \sum_{i,j=1}^n \rho(d_{\mathcal{S}}(s_i, s_j), d_{\mathcal{T}}(t_{l_i}, t_{l_j})) \\ &+ \lambda \sum_i \|s_i - t_{l_i}\| \end{aligned}$$

where $\|s_i - t_{l_i}\|$ disambiguates intrinsic symmetry

Implementation

- Preprocessing
 - Poisson reconstruction for geodesic distance
 - farthest point sampling
- Global optimization
 - sample hundreds of points
 - random permutation of the nodes for best solution
- Upsampling and refinement (optional)
 - upsample mapping to thousands of correspondences
 - refine the correspondences by fusion moves

FAUST Dataset



FAUST [Bogo et al. 2014]

Results

- Our approach outperforms a large body of prior work by a factor of 3

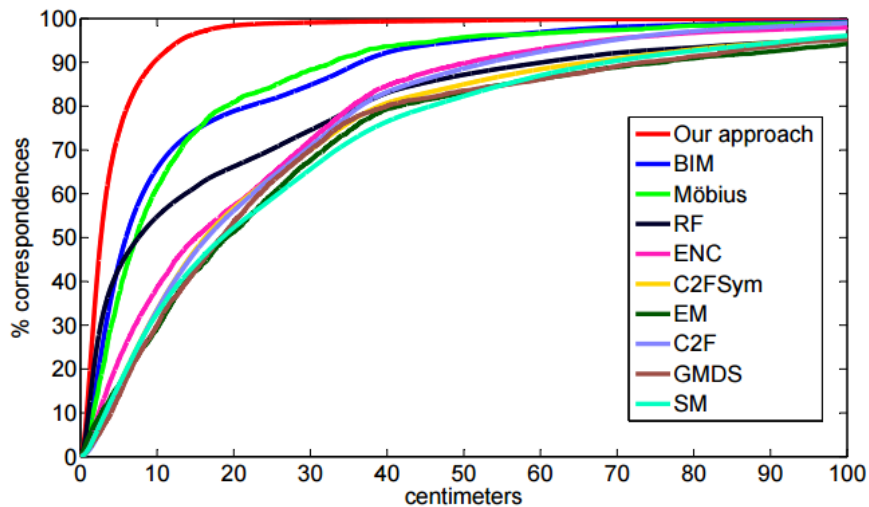
method	AE (cm)	worst AE	10cm-recall
GMDS [8]	28.98	91.84	0.300
Möbius [24]	14.99	80.40	0.614
BIM [19]	13.60	83.90	0.658
C2F [36]	23.63	73.89	0.333
EM [37]	30.11	95.42	0.293
C2FSym [38]	26.87	100.23	0.335
SM [30]	28.81	68.42	0.326
ENC [34]	23.60	51.32	0.385
RF [33]	22.26	69.26	0.548
Our approach	4.49	10.96	0.907

Accuracy on intra-subject pairs

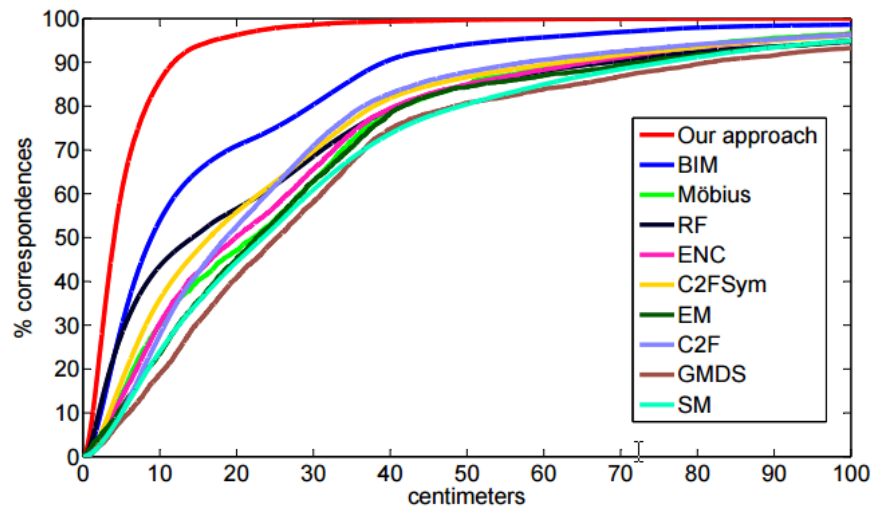
method	AE (cm)	worst AE	10cm-recall
GMDS [8]	35.06	91.21	0.188
Möbius [24]	30.58	70.02	0.300
BIM [19]	17.36	86.76	0.539
C2F [36]	25.51	90.62	0.277
EM [37]	31.25	90.74	0.235
C2FSym [38]	25.89	96.46	0.359
SM [30]	32.66	75.38	0.240
ENC [34]	29.29	57.28	0.303
RF [33]	26.92	79.43	0.435
Our approach	5.95	14.18	0.858

Accuracy on inter-subject pairs

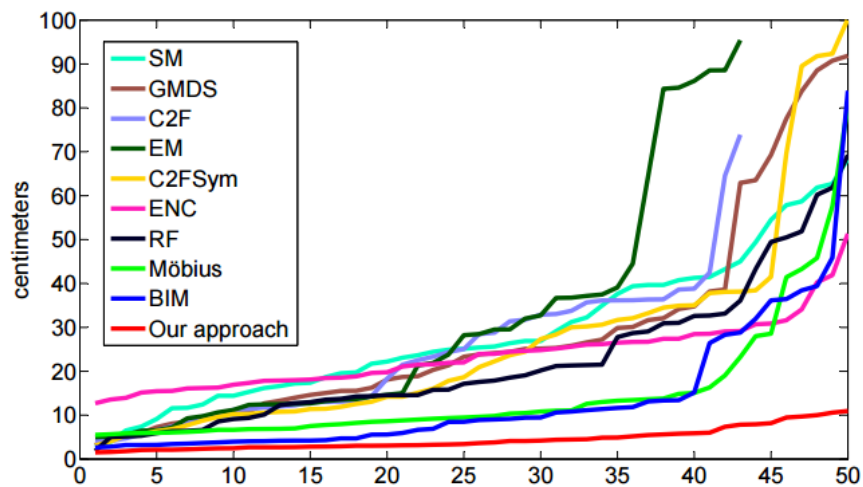
Results



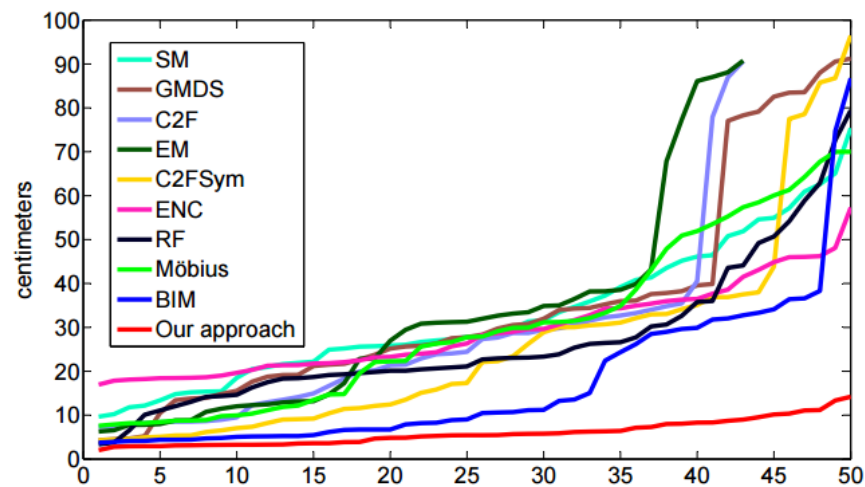
(a) Cumulative error distribution, intra-subject



(b) Cumulative error distribution, inter-subject

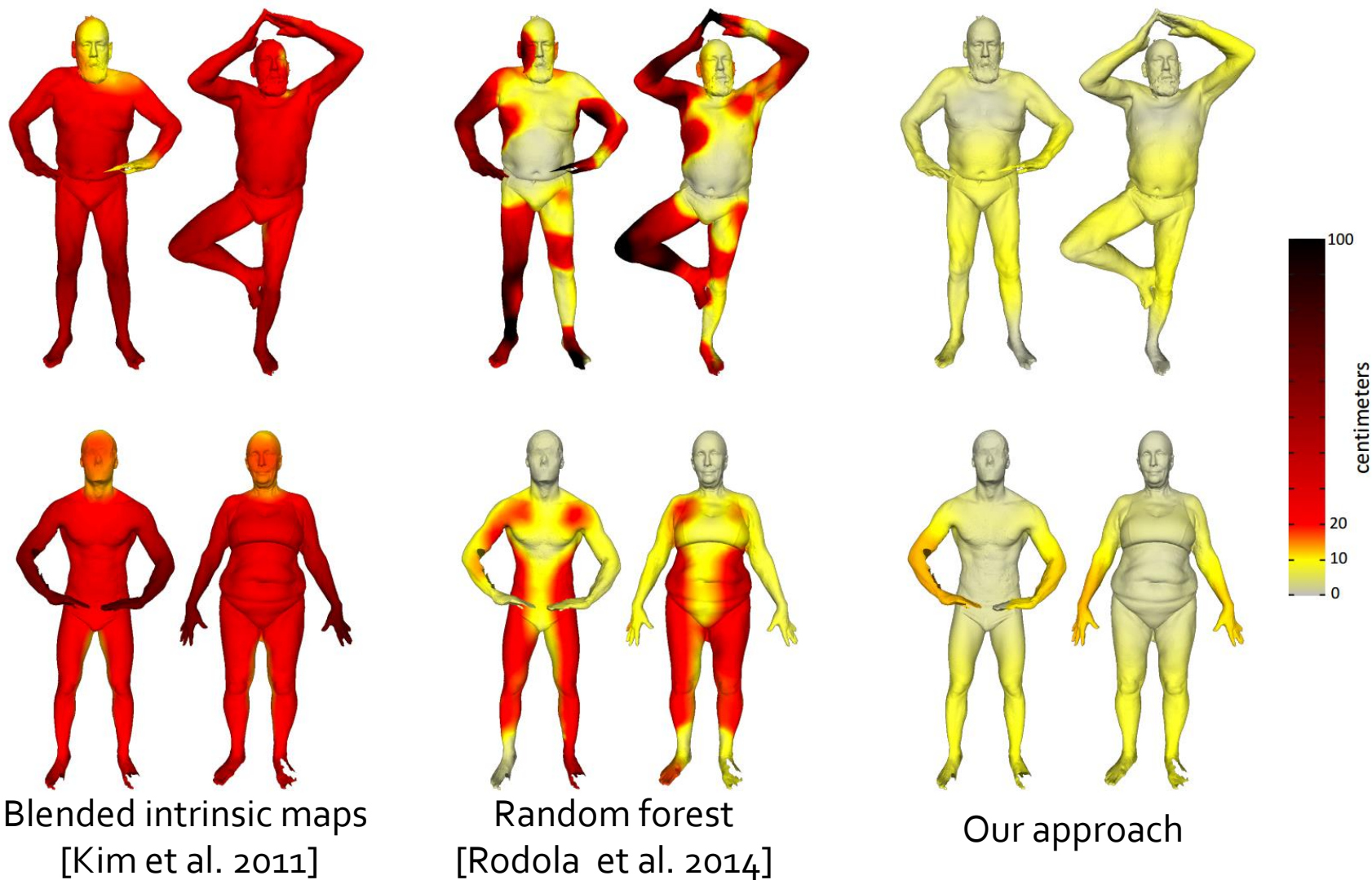


(c) Average error for each intra-subject pair



(d) Average error for each inter-subject pair

Results



Summary

- Simple but robust
 - no descriptors
 - convex optimization
 - outperforms a large body of prior work by a multiplicative factor
- Future work
 - partial surface registration
 - joint analysis of non-isometric shapes

Questions?

- Matlab, C++ code, and data
<http://www.stanford.edu/~cqf/convex/>